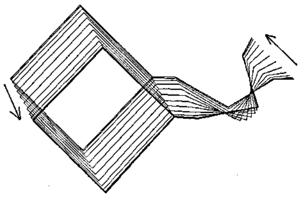
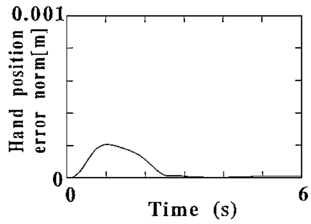
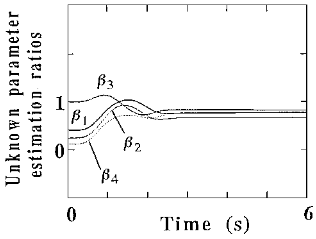


a) Robot motion

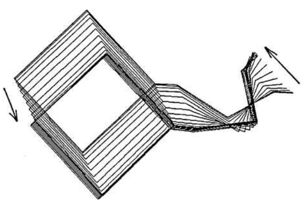


b) Hand position error

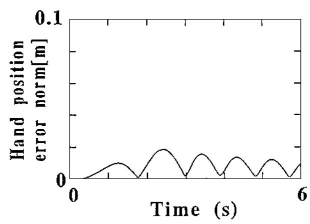


c) Parameter estimation

Fig. 1 Control result of adaptive control (K_D = diag[6.0 6.0 6.0 6.0 6.0 6.0], Λ = diag[160 160 20 160 160 20], Γ = diag[0.001 0.1 1.0 1.0]).



a) Robot motion



b) Hand position error

Fig. 2 Control result of RAC (G_P = diag[160 160 120 160 160 120], G_D = diag[3.0 3.0 2.0 3.0 3.0 2.0]).

to the real values. Figure 2 is for the RAC. The figures show that the adaptive controller achieves better performance than the RAC. Hence, the adaptive controller is more effective than the RAC when the space robot manipulates an unknown payload. The estimated parameters tend to converge to the real values during the motion for 3 s as illustrated in Fig. 1c, whereas the convergence of the estimation is not guaranteed. The parameters converge to the real values when the robot repeats the same motion although the results are not shown here. No more simulation results are illustrated whereas the same results are obtained in some other cases where the position and/or the orientation of the satellite vehicle is not controlled.

Conclusion

The adaptive controller has been proposed for space robots manipulating payloads. The controller has been able to control all manipulation variables of the position and the orientation of the satellite and those of the hand. The small modification has allowed the controller to select any parts of the manipulation variables. The asymptotic stability of the closed-loop system was studied by Lyapunov's second method. The effectiveness of the controller has been examined by numerical simulations.

Acknowledgment

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Initial Adjoint-Variable Guess Technique and Its Application in Optimal Orbital Transfer

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Introduction

IT is well known that the main difficulty of the indirect method for trajectory optimization is the requirement for an educated guess of the initial adjoint variables. A number of papers have been proposed to deal with the guess. Perhaps the adjoint-control transformation method presented by Dixon[1] is the most efficient and attractive. This Note also investigates the adjoint-variable guess problem using the control variables. The approximate initial adjoint variables are obtained by solving equations in the neighborhood of the initial time in which the adjoint variables can be expressed by a first-order Taylor series expansion. Two numerical examples are presented for optimization of very low thrust trajectories.

Optimal Control Problems

Consider the state equations that are not explicit functions of time

$$\dot{x} = f[x(t), u(t)] \tag{1}$$

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and the boundary conditions

$$\Psi(x_f, u_f) = 0 \tag{2}$$

where the initial conditions

$$x(0) = x_0 \tag{3}$$

at the fixed initial time t_0 are given. The control variable $u(t)$ must be select to minimize the performance index

$$J = \phi(x_f) \tag{4}$$

subject to the constraint conditions (1–3).

The variational method is used to solve the problem. Let us define the Hamiltonian

$$H = \lambda^T f[x(t), u(t)] \tag{5}$$

and the auxiliary function

$$\Phi = \phi + \nu^T \Psi \tag{6}$$

The necessary conditions for optimal control are as follows:

$$\frac{\partial H}{\partial u} = 0 \tag{7}$$

called the optimal control equation,

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \tag{8}$$

called the adjoint equation,

$$\lambda(t_f) = \left.\frac{\partial \Phi}{\partial x}\right|_{tf} \tag{9}$$

called the transversality conditions, and if the final time t_f is free, we still have

$$\left.\left(\frac{\partial \Phi}{\partial t} + H\right)\right|_{tf} = 0 \tag{10}$$

where λ are adjoint variables and ν are Lagrange multipliers.

Numerical solutions for the optimal control problems result in a two-point boundary-value problem (TPBVP). To solve the TPBVP, we must choose good initial adjoint variables.

Initial Adjoint Variable Guess

In the neighborhood of the initial time, the adjoint variables can be written as

$$\lambda_n = \lambda_0 + \dot{\lambda}|_{t_0} t_n \qquad (n = 1, 2, \ldots, N) \tag{11}$$

where t_n is the time in the neighborhood of t_0 and N is the number of unknown initial adjoint variables. As the state equations do not include time explicitly, the Hamiltonian is constant, i.e.,

$$H = c \tag{12}$$

With the aid of Eqs. (5), (8), and (11), we have

$$\left(\lambda_0 - \left.\frac{\partial H}{\partial x}\right|_{t_0} t_n\right)^T f_n = c \tag{13}$$

Because control variables have explicit physical meanings, we could make a good guess about $u(t)$ by empirical or trial-error methods. So, f_n can be obtained approximately by integrating Eq. (1) under the initial condition equation (3).

The constant c can be set to an arbitrary value, except zero, if the final time is fixed because the optimal control solution is unchanged due to the common scaling.[2] The c also can be obtained by Eq. (10) if the final time t_f is free. When the time points $t_1, t_2, \ldots, t_N$ are selected in the neighborhood of t_0, the linear equations whose dimension is N can be established according to Eq. (13). The approximate initial adjoint variables can be obtained by solving the equations.

Numerical Examples

The two examples for very low thrust trajectory optimization that have been investigated by the direct approach in Scheel and Conway[3] are computed. The performance index is the maximum final mass. The directions of the guessed control variables are set tangential for very low thrust trajectories.

Equations of Motion and Requirements

The system equations are established in a polar coordinate frame:

$$\dot{v}_r = \left(v_\theta^2 / r\right) - (\mu / r^2) + (T/m)\sin u \tag{14}$$

$$\dot{v}_\theta = -(v_r v_\theta / r) + (T/m)\cos u \tag{15}$$

$$\dot{r} = v_r \tag{16}$$

$$\dot{\theta} = v_\theta / r \tag{17}$$

$$\dot{m} = -(T/I_{sp}) \tag{18}$$

The states of the system are radial position r, radial velocity v_r, circumferential velocity v_θ, polar angle θ, and mass of the spacecraft m. The gravitational parameter for Earth is denoted by μ. The thrust-direction angle u is measured from the local horizon and is defined positive above the horizon in the direction of the spacecraft's motion. T is the constant magnitude of the thrust, and I_{sp} is the specific impulse.

Example 1

Example 1 is low Earth orbit (LEO)-geostationary Earth orbit (GEO) transfer. According to Ref. 3, the spacecraft starts in a circular LEO and is required to transfer to a coplanar (circular) GEO in an unspecified final time. The initial LEO altitude is chosen to be 270 km, or 1.0423 DU for the initial semimajor axis in canonical units (1 DU, distance unit, is equivalent to the Earth's radius and the gravitational parameter μ is 1 DU^3/TU^2 in canonical units where TU is a time unit). The final semimajor axis at GEO will be 6.63 DU. The thrust acceleration is 325 μg, and the specific impulse is 2.0 in normalized units.

Because the constant c is equal to zero from Eqs. (10) and (12), the mass adjoint variable λ_m can be arbitrarily selected so that Eq. (13) will be solved. The time points are separated by 10 s. The number N is 3. The hybrid direct/indirect optimization approach[4,5] is used here, and the optimization algorithm is sequential quadratic programming. The maximum final mass of the index is equal to the minimum final time for continuous constant thrust, and the approximate final time can be obtained by integrating the tangential thrust path whose final altitude reaches GEO. The results are given in Table 1. The minimum time or transfer time is slightly better than that of Ref. 3 (1579.16 TU).

Example 2

Example 2 deals with orbit raising from GEO. According to Ref. 3, the spacecraft starts in a circular geosynchronous orbit and is required to transfer to another coplanar orbit (circular) of a specified radius in an unspecified final time. In canonical units the semimajor axis at GEO is 6.63 DU. The final semimajor axis is selected to be 10 DU. The thrust acceleration is 4.95 μg, and the specific impulse is 2.0 in normalized units. The corresponding results are given in Table 2. The minimum time is also slightly less than that of Ref. 3 (14314.5 TU).

Table 1 Solution of initial adjoint variables for LEO-GEO transfer

Status	λ_{vr}	$\lambda_{v\theta}$	λ_r	λ_m	H	T_f
Guess	−0.223	−500.0	−470.0	−1000.0	0	1575.94
Exact	−12.0	−507.3	−465.6	−1000.0	−0.0025	1578.93

Table 2 Solution of initial adjoint variables for orbit raising from GEO

Status	λ_{vr}	$\lambda_{v\theta}$	λ_r	λ_m	H	t_f
Guess	−1.37	−5000.0	−293.0	−10000.0	0	14304.64
Exact	−10.1	−5000.1	−290.7	−10000.0	$-6.6e-7$	14314.34

Conclusion

The initial adjoint-variable guess technique has been researched in this Note. All of the approximate initial adjoint variables can be obtained. The minimum-time orbital transfers for very low thrust are solved by using this technique, and the results are slightly better than those of the direct method. The numerical examples demonstrate that the technique is effective.

Acknowledgment

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Identifying Helicopter Control Gains Using Desired Output Histories

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Introduction

A NEW method is presented for synthesizing control feedback and feedforward gains using desired output histories for the given inputs. Conventional design techniques use parameterized design criteria, and the time response of the closed-loop system is evaluated after the control gains are computed. However, this technique directly uses pairs of input and output time histories to find control system parameters. The control gains are computed so that the closed-loop outputs are as close as possible to the user-specified ones for the given inputs and is viewed as an identification process. Among the various identification methods, this study uses a method based on the smoothing technique as proposed by Idan and Bryson.[1]

This method is computationally intensive, but it is possible to implement this algorithm with modern desktop computers. It can be applied to many cases: simple stability augmentation system design, command following logic design, gain scheduling, etc. As an example, the new method is applied to a helicopter control system design problem.

Design with an Identification Method

Identification problems are inherently nonlinear and must be solved iteratively. At each iteration step, the input and output data are smoothed using the current set of parameters. In the process of smoothing, adjoint variables, which are the sensitivities of the performance index to the state variables, are computed. The sensitivities of the performance index to the system parameters are then computed using the adjoint variables. The system parameters are updated using these sensitivities at the end of each iteration step. The process is repeated until the changes in the system parameters become negligible.

The smoothing procedure is described as follows. Given a discrete linear state-space model

$$\boldsymbol{x}(i+1) = \Phi(\Theta)\boldsymbol{x}(i) + \Gamma(\Theta)[\boldsymbol{u}_0(i) + \boldsymbol{w}(i)] \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{x}(i) + \boldsymbol{D}[\boldsymbol{u}_0(i) + \boldsymbol{w}(i)] \tag{2}$$

and a pair of input and desired output histories

$$\boldsymbol{u}_0(i), \qquad i = 1, \ldots, N-1$$
$$\boldsymbol{y}_d(i), \qquad i = 1, \ldots, N$$

determine the control parameter set Θ that minimizes the performance measure:

$$J = \frac{1}{2}[\boldsymbol{x}(0) - \boldsymbol{x}_0]^T \boldsymbol{S}_0[\boldsymbol{x}(0) - \boldsymbol{x}_0] + \frac{1}{2}[\boldsymbol{x}(N) - \boldsymbol{x}_f]^T \times \boldsymbol{S}_f[\boldsymbol{x}(N) - \boldsymbol{x}_f] + \frac{1}{2}\sum_{i=0}^{N-1}\Big\{\boldsymbol{w}^T(i)\boldsymbol{R}\boldsymbol{w}(i) + [\boldsymbol{y}(i+1) - \boldsymbol{y}_d(i+1)]^T \boldsymbol{Q}[\boldsymbol{y}(i+1) - \boldsymbol{y}_d(i+1)]\Big\} \tag{3}$$

where $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{S}_0$, and $\boldsymbol{S}_f$ are weighting matrices; $\boldsymbol{w}$ is a perturbation input; and $\boldsymbol{x}_0$ and $\boldsymbol{x}_f$ are guesses for the initial and final conditions. This is the most general form of the performance index. If test data are used as the input and output pairs, the noise in the measurement can be handled by using this comprehensive form. For deterministic cases, setting the weightings on the unnecessary terms, such as $\boldsymbol{w}$, to large numbers makes the effects of those terms negligible in the final results. The system matrices (Φ and Γ) are functions of control parameters but are treated as constant matrices for each iteration step. The optimization problem is to minimize the following augmented performance index with respect to $\boldsymbol{x}$ and $\boldsymbol{w}$:

$$\bar{J} = J + \sum_{i=0}^{N-1}\boldsymbol{\lambda}^T(i+1)\{\Phi(\Theta)\boldsymbol{x}(i) + \Gamma(\Theta)[\boldsymbol{u}_0(i) + \boldsymbol{w}(i)] - \boldsymbol{x}(i+1)\} \tag{4}$$

where $\boldsymbol{\lambda}$ is the set of Lagrange multipliers for the state equations. For the solution to this minimization problem, see Idan and Bryson.[1]

The derivatives of the performance index with respect to the control parameters are computed as follows:

$$\frac{d\bar{J}}{d\Theta} = \sum_{i=0}^{N}\boldsymbol{\lambda}^T(i+1)\left\{\frac{\partial\Phi(\boldsymbol{K})}{\partial\Theta}\boldsymbol{x}(i) + \frac{\partial\Gamma(\boldsymbol{K})}{\partial\Theta}[\boldsymbol{u}_0 + \boldsymbol{w}(i)]\right\} \tag{5}$$

Equation (5) is for only one pair of inputs and outputs. As the complexity of the system model increases, more than one pair of inputs and outputs may be needed to guarantee the uniqueness of the solution. For the case of multiple sets of inputs and outputs, the performance index is defined as

$$J = \sum_{s=1}^{n\,\mathrm{set}} w_s(s) J_s \tag{6}$$

where n set is the number of input/output pairs and $w_s(s)$ represents the weighting on the sth data set. For each iteration step, the program solves the optimization problem for each set independently. Once the derivatives are computed for all sets, the effective derivative is computed as

$$\frac{d\bar{J}}{d\Theta} = \sum_{s=1}^{n\,\mathrm{set}} w_s(s)\frac{d\bar{J}_s}{d\Theta} \tag{7}$$

At the end of each iteration step, the control parameters are updated using this derivative information. This study uses a conjugate gradient method to find the control parameters.

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